The University of Melbourne School Mathematics Competition, 2026

INTERMEDIATE DIVISION
Solutions
Time allowed: Three hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The* ***seven*** *questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

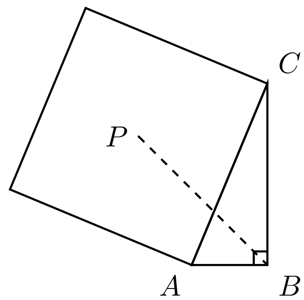
Textbooks, electronic calculators and computers are ***NOT*** *allowed. Otherwise normal examination conditions apply.*

1. Determine the number of solutions to the equation

$$x_1 + x_2 + \cdots + x_7 = 12,$$

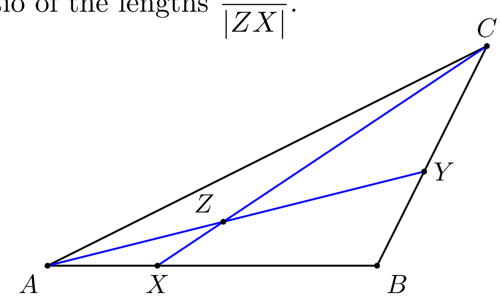
in integers $x_1, x_2, \ldots, x_7$, where $0 \leq x_i \leq 2$ for $i = 1, 2, \ldots, 7$.

2. Point P lies at the centre of the square with side the hypotenuse of a right-angled triangle ABC as in the diagram. What is the value of the angle $\angle PBC$? Prove your claim.



3. A bucket contains more than one each of balls coloured red, blue, green and yellow. You are to be blindfolded and draw out four balls and you win if the four balls are of different colours. Before beginning, one blue ball in the bucket is replaced by a red ball, and your probability of winning increases to 4/3 of what it was before. How many blue balls are now in the bucket?

4. In a triangle ABC, points X and Y are on sides AB and BC respectively such that the interval lengths are $|AX| = 1$, $|XB| = 2$, $|BY| = 3$, $|YC| = 4$. Let Z be the intersection of CX and AY. Determine the value of the ratio of the lengths $\dfrac{|CZ|}{|ZX|}$.



5. Find all real numbers x which satisfy the inequality

$$\sqrt{3 - x} - \sqrt{x + 1} > 1.$$

6. The numbers 4, 5, 6, 7, 8, 9 are placed on different faces of the cube. Starting from any face, build a 3 digit number by traveling to adjacent faces without returning. So if you travel through faces with 4, then 9 then 6, then the number is 496. Do this for all three step paths to produce a collection of three digit numbers. One of these three digit numbers is a factor of the sum of the others. What is this number?

7. Three numbers a, b and c are chosen at random, subject to the conditions that $a + b + c = 1$ and that a, b and c are the side lengths of a triangle. Which is more likely, that the triangle with side lengths a, b and c is acute or obtuse? Prove your claim.