The University of Melbourne School Mathematics Competition, 2026

SENIOR DIVISION

Time allowed: Three hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **seven** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. At the Colac cricket club, players get paid according to how many runs they score. There are two cutoffs, the good score cutoff and the great score cutoff (which is higher than the good score cutoff). A score higher than the great score cutoff earns a certain amount of money, and a score between the good and great score cutoffs earns a different higher amount of money.

Connie earns 100 dollars with scores of 64 and 77.

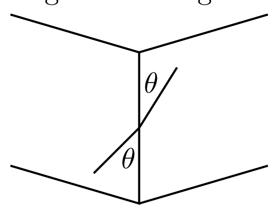
Charlie earns 280 dollars with scores of 15, 42, 80 and 108.

Chris earns 240 dollars with scores of 2, 23, 40, 55 and 92.

How much money is earned for a great score and how much money is earned for a good score?

2. Given any four distinct non-zero digits a, b, c, d, let $n_1, n_2, \ldots, n_{24}$ be an enumeration of all possible 4-digit numbers containing the digits a, b, c, d once each. Let D be the greatest common divisor of $|n_1 - n_2|$, $|n_2 - n_3|$, $\ldots$, $|n_{23} - n_{24}|$. What is the largest possible value of D over all choices of a, b, c, d and $n_1, n_2, \ldots, n_{24}$?

3. An ant walks along the faces of a cube. It always walks along a straight path, so that the angle it forms with any edge is the same on entering and exiting as in the diagram.



Show that it is possible for the ant to start at a point on the cube, walk along a straight path avoiding all vertices and meeting every face, and only meeting its path again at its starting position.

4. Let $\{a_n\}$, $n = 1, 2, ...$ be a strictly increasing sequence of non-negative integers with $a_2 = 4$. Prove that there exists an $m > 1$ such that

$$a_m^2 \leq 2a_{m^2}.$$

5. A plane intersects a regular tetrahedron with side length L in a rectangle. Find the largest possible area of intersection.

6. Show that the polynomial

$$P(m, n) = m^3 - 3m^2n - 6mn^2 + 8n^3$$

is never equal to 2026, no matter which integers m and n are chosen.

7. Sinead and Dominic are playing scissors-paper-rock with a twist. Sinead is never allowed to choose paper and Dominic is never allowed to choose rock. (So, at each turn Sinead and Dominic choose to play rock, paper or scissors, announce their decision simultaneously, and scissors beats paper beats rock beats scissors determines the winner, or a tie if both choose the same.) They agree to play a series of at most 2026 games until either someone wins a game, or 2026 games have been tied. In the latter case, Dominic will be declared the winner. With perfect play, what is the probability that Sinead wins?