The University of Melbourne School Mathematics Competition, 2025

INTERMEDIATE DIVISION

Time allowed: Three hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. Two ladies Mrs. A and Mrs. B live in separate towns. At the same time one day, each sets out on foot for the town of the other, walking at her own constant rate. They pass at noon. Mrs. A reaches her destination at 4 pm, while Mrs. B reaches her destination at 9 pm. At what time did they set out?

2. Let p, q be consecutive prime numbers greater than 2. Show that $p + q$ is a product of three integers, each greater than 1. (The only factors of a prime number are 1 and itself, and an example of consecutive prime numbers is 23 and 29.)

3. Consider a regular decagon with sides of length 1cm inscribed inside a circle as in the left diagram. Inscribe a second circle inside the decagon. What is the area between the two circles? It is shaded in the right diagram.
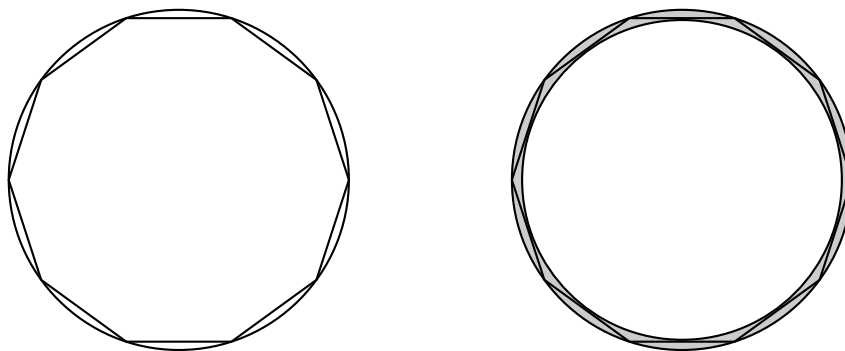
4. Find all integer solutions of $x^3 + y^3 = 2025(x + y)$.

5. An elevator starts with 10 people on board and can stop at 10 floors. Each person picks a floor uniformly at random (independently). What's the expected number of floors that the elevator stops at?

6. The diagram below shows points A and B on a horizontal line, and a circle containing these points. For any point P on the circle, let P' be the point directly below P such that the line through B and P' meets the line through A and P at right angles. Describe the path that P' takes as P moves around the circle.

Hint: It may help to show that B lies on the path taken by P'.