The University of Melbourne School Mathematics Competition, 2025

JUNIOR DIVISION

Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. In the following sum, each letter represents a distinct digit. What is the largest possible value for the number MATHS?

$$\begin{array}{r} \text{VOTE} \\ +\text{ONE} \\ \hline \text{MATHS} \end{array}$$

2. Five people are sitting around a table and they have a deck of four cards, each with a positive integer on them. Alice draws two cards, looks at them and announces "The sum of my two numbers is 16" before returning the cards. Bob then draws two cards, looks at them and announces "The sum of my two numbers is 3" before returning the cards. Carol then draws two cards, looks at them and announces " The sum of my two numbers is 32" before returning the cards. Dwayne then draws two cards, looks at then and announces "The sum of my two numbers is 22". Erin then draws two cards, looks at them and announces "The sum of my two numbers is 8." Is it possible they are all correct about the sum of the number of the cards they drew?

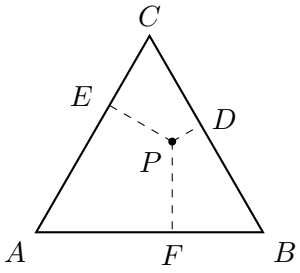
3. Find the largest positive integer n such that $n^2 + 20n + 25$ is a perfect square.

4. Some perfect fifth powers have all their digits distinct, like $5^5 = 3125$. Some do not, like $6^5 = 7776$. Show that at most 89 perfect fifth powers of positive integers have all their digits distinct.

5. Do there exist two tetrahedra (triangular pyramids) S and T satisfying the following?

(1) The volume of S is smaller than the volume of T.
(2) The area of each face of S is larger than the area of any face of T.

6. Let ABC be an equilateral triangle of side length 1. Let P be a point in the interior of triangle ABC. Let D, E and F be the feet of the perpendiculars from P to each of BC, CA and AB respectively.



Find the value of the sum of the lengths of the three perpendiculars, shown as broken lines,

$$|PD| + |PE| + |PF|.$$