The University of Melbourne School Mathematics Competition, 2024

JUNIOR DIVISION

Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The* ***six*** *questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

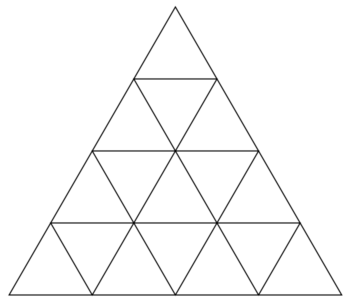
(4) *You may need to do rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

Textbooks, electronic calculators and computers are ***NOT*** *allowed. Otherwise normal examination conditions apply.*

1. On her birthday in 2024, Taylor's age in years will be equal to three times the sum of the digits in the year she was born. In what year was Taylor born?

2. The below picture is constructed out of 16 equilateral triangles each of the same side length.



There are fifteen vertices in this picture. From these fifteen points, how many sets of three points X, Y and Z are there such that the triangle XYZ is equilateral?

3. Beth thinks that any composite number which is divisible by 2, 3, 5 or 11 is *obviously composite*, since she can easily check divisibility by these numbers, and calls any composite number not divisible by 2, 3, 5 or 11 *non-obviously composite.* If a positive integer between 1 and 200 inclusive is picked at random, what is the probability that it is non-obviously composite?

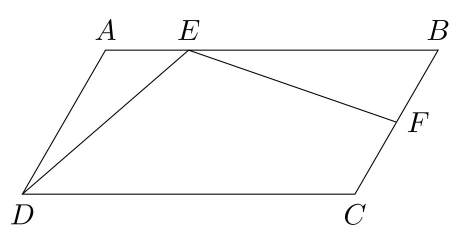
(A positive integer is said to be composite if it has a divisor other than 1 and itself.)

4. Meadowbrook and Willow Springs are two neighbouring towns. In Meadowbrook the average age is 30 and in Willow Springs, the average age is 40.

(a) Show how it is possible for someone to move from Willow Springs to Meadowbrook so that the average age of both towns increases.

(b) Is it possible for someone to move from Willow Springs to Meadowbrook so that the average age of both towns increases, and then for someone to move from Meadowbrook to Willow Springs so that the average age of both towns increases again?

5. Let $ABCD$ be a parallelogram. Let E be the point on the line segment AB such that $|BE| = 3|AE|$. Let F be the midpoint of BC. If the area of the parallelogram $ABCD$ is 2024, what is the area of the quadrilateral $CDEF$?



6. Jack and Jill are playing a game, which consists of w white balls, 2 black balls, and b buckets, where $w \geq b \geq 2$ are positive integers. Without Jill watching, Jack puts all the balls into the buckets in such a way that no bucket is empty. Then Jill chooses a bucket at random, and then chooses a ball from that bucket at random. Jill wins if she chooses a white ball. How should Jack distribute the balls in order to maximise the chances of Jill winning, and what is Jill's probability of winning if Jack distributes the balls in this way?