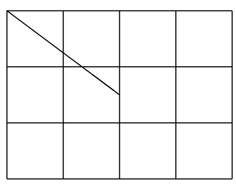
1. The largest possible value obtainable when a 1-digit number is multiplied by a 2-digit number is $9 \times 99 = 891$, so the output of Melissa's multiplication has at most 3 digits. Since the digit 3 is not shown, the possibilities are 21, 321, 231 and 213. The prime factorisations of each of these numbers are $21 = 3 \times 7$, $321 = 3 \times 107$, $231 = 3 \times 7 \times 11$ and $213 = 3 \times 71$. We examine each in turn to find out which of these numbers have factorisations involve a 1-digit number times a 2-digit number. The answers are $21 = 1 \times 21$, $231 = 3 \times 77 = 7 \times 33$ and $213 = 3 \times 71$. All the possible products except the first are not possible for Melissa to input as she is missing the button 3. Therefore she must be multiplying 1×21.

2. We will show the answer is $6,219$. The fourth digit is at most 9, and the second and third digits have to sum to at least 3, since they are distinct and not zero. Therefore the first digit is at most $9 - 3 = 6$. In order to have the first digit 6, we must have equality everywhere, therefore the second and third digits are 1 and 2 in some order, and the fourth digit is 9. The largest of these options is the number $6,219$, which we can easily check is indeed funny.

3. After each child takes their share, the amount of the farmer's estate that remains is reduced by three-quarters. So if there are n children, then the fraction of the farmer's estate donated to charity will be $(3/4)^n$. The oldest child gets $1/4$ of the estate, so we seek the largest value of n for which $(3/4)^n > 1/4$. This is equivalent to $3^n > 4^{n-1}$. If $n = 4$, then this inequality is $81 > 64$ which is true, while if $n = 5$, this inequality is $243 > 256$ which is false. Therefore the maximum number of children the farmer can have is 4.

4. Because at most 5 of the digits are larger than 3, there can be at most one occurrence of 6, 7 and 8 in a row. If these digits occur in the 2nd, 3rd and 4th positions, then the first digit can be any of 0,1,2,3 and 4, while the other two digits are unrestricted. Therefore there are $4 \times 10 \times 10 = 400$ numbers of the form $*67,8**$. There are two other locations where the digits 6, 7 and 8 can occur, and similarly there are 400 possibilities for each of these cases. So in total, there are $3 \times 400 = 1200$ numbers satisfying the given conditions. The probability is therefore
$$\frac{1200}{400000} = \frac{3}{1000}.$$

5. The answer is 6.



The picture above shows one side of the box. Half of the diagonal is shown, which represents the part of the line between opposite egdes that lies in the $1 \times 3 \times 4$ sub-box. Inside this sub-box, we count and see there are 3 cubes interesected, so in total there must be $2 \times 3 = 6$.

6. We have
$$|AEF| = \frac{1}{3}\frac{2}{3}|ABC| = \frac{20}{3}$$
since the height differs by a factor of 1/3 and the base differs by a factor of 2/3 (thinking of the side along AC as the base).

   Similarly $|CDE| = |BFD| = \frac{20}{3}$.

   Therefore
$$|DEF| = |ABC| - |AEF| - |CDE| - |BFD| = 30 - 3 \times \frac{20}{3} = 10.$$

7. Consider the green string attached to the westmost house on the north side of the street. It is attached to some house, call it A on the south side of the street. The red string attached to A cannot cross this green string attached to A, so must also be connected to the westmost house on the north side of the street. So the two strings from the westmost house on the north side of the street must be connected to the same house on the south side of the street.

   Once we know this, we can apply a similar argument to show that the two strings from the middle house on the north side of the street are connected to the same house on the south side of the street, and again for the last house.

   This leaves us with the question of how many ways we can match up houses from the north side of the street to the south side, which is $3! = 6$. (Each such matching does lead to a valid configuration of lights).