The University of Melbourne School Mathematics Competition, 2023

JUNIOR DIVISION

Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **seven** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. Melissa has a calculator that is missing the button 3. In addition, the display refuses to display the number 3, omitting that digit whenvever the result of a computation should involve the number 3. For example when 4×59 is computed, the output is shown as 26. Melissa multiplies a 1-digit number and a 2-digit number together on her calculator, and is shown the result of 21. What are all the possibilities for the numbers Melissa is multiplying?

2. A number is called funny if all of its digits are different, it does not contain the digit zero, and its last digit is the sum of all of its other digits. What is the largest funny number?

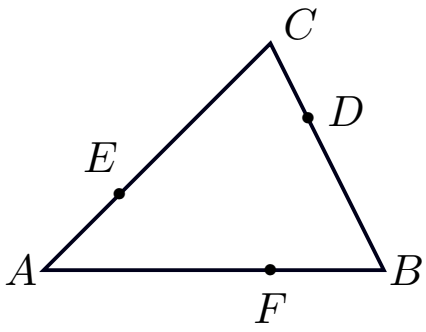
3. A farmer is writing his will and needs to divide his estate amongst his children. The farmer decides that the oldest child gets one-quarter of the farmer's estate, then the next oldest child gets one-quarter of what remains, then the next oldest gets one-quarter of what remains, and so on, until all children have received their share. Anything left over after this process is donated to charity. It turns out that more money is donated to charity than goes to the oldest child. What is the maximum number of children the farmer can have?

4. An integer is chosen at random between 0 and 399,999 inclusive. What is the probability that the number chosen contains the consecutive digits 6, 7 and 8 in that order? (e.g. 216,789 satisfies the condition, but 126,873 and 161,718 do not).

5. A rectangular box of side lengths 2, 3 and 4 is divided into 24 unit cubes. How many of these 24 unit cubes intersect the line drawn between two opposite vertices?

6. Let ABC be a triangle. Points D, E and F are chosen on sides BC, CA and AB respectively such that

$$\frac{|BD|}{|DC|} = \frac{|CE|}{|EA|} = \frac{|AF|}{|FB|} = 2.$$



If the triangle ABC has area 30, what is the area of triangle DEF?

7. There is a street running east-west, with 3 houses on each side. The occupants of the houses want to put up some strings of red and green Christmas lights across the street, according to the following rules:

- Each string starts and finishes at a house, crosses the street in a straight line, and is of a single colour.
- Each house has one red string and one green string attached to it, with the red string attached at a point to the west of the green string.
- The two strings originating from the same house on the southern side of the street may not cross

How many ways can they do this?