The University of Melbourne School Mathematics Competition, 2022

INTERMEDIATE DIVISION

Time allowed: Three hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **seven** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

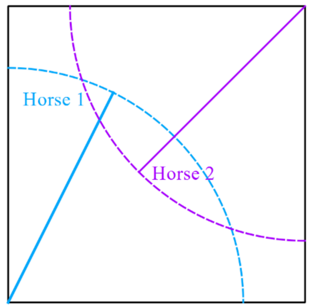
*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. Recall that

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1.$$

How many zeros are at the end of the number 50!?

2. A square field contains two horses, attached by ropes to diagonally opposite corners of the field. Both ropes are of equal length, such that each horse can reach half the field. What is the difference between the area of the field that both horses can reach and the area that neither horse can reach?

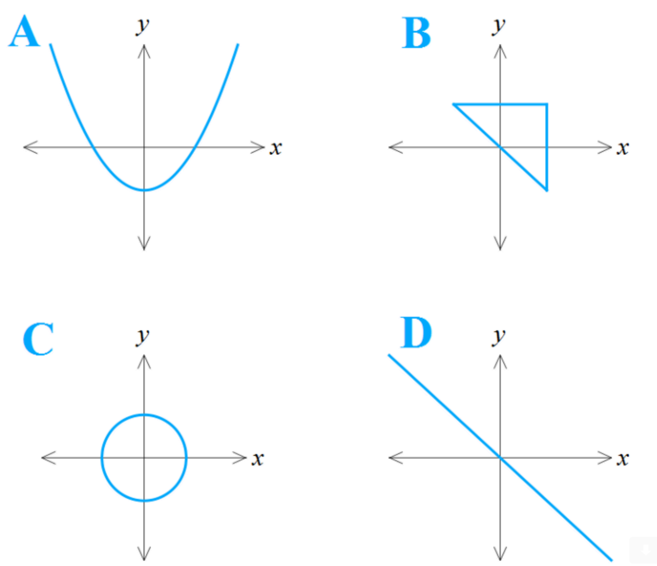


3. Calculate the sum

$$\sin^2(1) + \sin^2(2) + \sin^2(3) + \cdots + \sin^2(90),$$

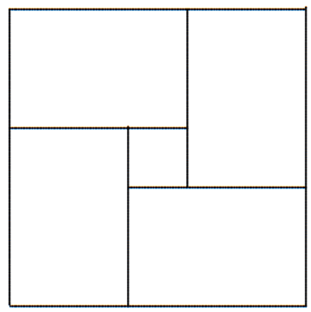
where the arguments are in degrees.

4. For which of the four following graph(s) can it be true that for each point (x, y) on the graph, $p(x) + p(y) = 0$ for some polynomial function p? (Recall that a polynomial can be a constant, or linear, as in $a + bx$, or quadratic, as in $ax^2 + bx + c$, or cubic etc.)



5. Suppose you wish to send a carefully chosen gift to each of eight friends. You wrap the gifts in parcels, and address them to the recipients. However, you become distracted, and end up forgetting which gift is in which parcel, so you address them at random, and hope that you are correct. What is the probability that only 1 of the eight is correctly addressed?

6. Four rectangles are arranged in a square pattern so that they surround a smaller square, as shown in the figure. Let O be the area of the outer square, and let I be the area of the inner square. If $O/I = 9 + 4\sqrt{5}$, determine the ratios of the lengths of the sides of the rectangles.



7. Find all the decimal numbers of the form 101, 10101, 1010101, etc (i.e. alternating 1s and 0s, starting and ending with a 1), that are prime numbers.