



The University of Melbourne School Mathematics Competition, 2022

JUNIOR DIVISION

Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

- (1) Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*
- (2) The **seven** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*
- (3) It may be necessary to spend considerable time on a problem before any real progress is made.*
- (4) You may need to do rough work but you should then write out your final solution neatly, stating your arguments carefully.*
- (5) Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

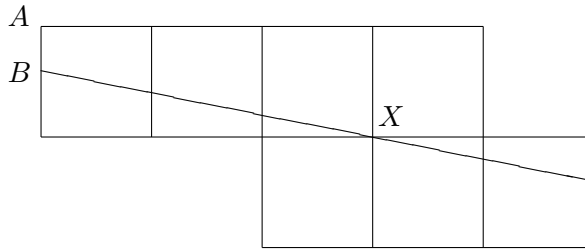
1. Trains leave Melbourne for Echuca on the hour every hour, and trains leave Echuca on the hour every hour. The journey takes 3 hours and 14 minutes in each direction. At 11am you catch a train to Melbourne from Echuca. How many Echuca bound trains do you pass before you reach Melbourne?

2. An integer is randomly chosen between 1 and 100 inclusive. What is the probability that it has no prime factors greater than 6?

3. Wilfred, Chantelle and Thelma are digging and find a buried treasure of 2022 gold coins. After much arguing, they agree on a method to divide the treasure amongst them. First Wilfred takes one coin. Then Chantelle takes two coins. Then Thelma takes three coins. Then Wilfred goes again and this time takes four coins. They continue taking turns like this, each time taking one more coin from the pile than the last person did on their previous turn. Who ends up with the most coins?

4. Sean goes to the toy shop and spends exactly \$44 on buying 22 toys. The toy shop sells three types of toys, trucks for \$9, cars for \$1 and motorbikes for 50c. His sister Sally tells him that she has enough information to know exactly how many of each type of toy Sean bought. How does Sally know this?

5. Po baked a cake that is comprised of seven $10\text{cm} \times 10\text{cm}$ squares, as shown in the diagram below. She now needs to cut it exactly in half with a straight line cut that goes through the central point X . The point A is the upper left corner of the cake and the point B is where the cut reaches the left edge. What is the length of AB ?



6. Recall that for an integer m , the factorial of m is defined as the product

$$m! = m \cdot (m - 1) \cdot (m - 2) \cdots 3 \cdot 2 \cdot 1.$$

Find all pairs of positive integers (m, n) which satisfy the equation

$$m! + 3 = n^2.$$

7. Pinocchio tells Geppetto that he can take a square, divide it into three triangles and rearrange them to form an equilateral triangle. How does Geppetto know that Pinocchio is lying?

Would Pinocchio still be lying if he said he could rearrange them to form some triangle?