The University of Melbourne–Department of Mathematics and Statistics
School Mathematics Competition, 2021
## JUNIOR DIVISION
Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. Not hard enough. You are competing in the ABC's game show "Hard Quiz", presented by comedian Tom Gleeson. With your topic of choice, *Mathematics*, you have reached the final head-to-head round and are competing for the limited edition Big Brass Mug. Tom likes to think he is smarter than the competitors on his show and with a particularly smug face asks you the following: "n comedians performed at this year's Melbourne Comedy Festival. No two comedians told the same number of jokes, no comedian told 2021 jokes, and no comedian told more than n or no jokes at all. What is the largest number of comedians that could have performed at the comedy festival?" Answer Tom's question, and wipe that smile off his face.

2. Now that's cricket. As a consequence of the COVID-19 pandemic and the three-month Greater Melbourne lockdown, fewer people are currently physicially active than before the pandemic struck. A special task-force of the state government led by Minister for Community Sport, Ros Spence, is currently looking at initiatives to get the population active again on the sporting field. One idea of the task-force aimed at boosting the number of year 7 and 8 students playing cricket is to make the game more exciting by changing the oval shape of the field to an irregular pentagon:
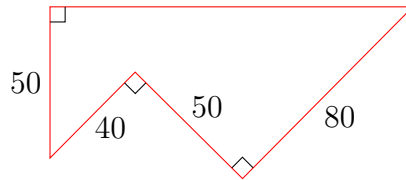

with dimensions measured in metres. What is the total area of the proposed new playing field?

3. Our proud COVID-19 record. Australia has been widely praised around the world for dealing with the COVID-19 crisis much better than most countries. However, this is not our only great COVID-19 feat. We have also topped the world-rankings in hoarding toilet paper during the crisis, and our toilet-paper wars have been receiving as much attention around the world as our low infection rates. At your local supermarket, n greedy customers buy up all the stock of toilet paper. The first customer buys 10 packs of toilet paper, plus one tenth of the remaining stock. The next customer buys 20 packs of toilet paper plus one tenth of the remaining stock. This continues until the nth customer buys $10 \times n$ packs of toilet paper plus one tenth of the remaining stock. The shelves are now completely empty. If all n greedy customers bought the same amount of toilet paper, how many packs of toilet paper did your local supermarket stock?

4. The devil to pay. A group of 64 people is assigned the numbers 1 to 64 by the number devil. Nobody in the group knows which number has been assigned to them or to anyone else in the group. The task of the 64 is to find out who has been assigned the numbers 1, 2 and 3. Any group of up to 8 people can ask the number devil to point out which person in that group has the lowest number, second lowest and third lowest number. The number devil will not, however, reveal what their actual numbers are. If a group of n people asks this question of the number devil it will cost them n dollars. In the best case scenario, what is the minimum total cost to the group of 64 to find out which person has been given the number 1, which person has been given the number 2 and which person has been given the number 3? In the worst case scenario, what is the minimum total cost to the group of 64 to find out which person has been given the number 1, which person has been given the number 2 and which person has been given the number 3?

5. Year of the ox. According to ancient Chinese folklore, oxen are hard workers, intelligent, reliable, and never demand praise. They are also the preferred prey of the Eurasian grey wolf. A Chinese oxherd has lost almost all of her oxen to wolves, and wishes to protect her few remaining oxen by building a 2-metre high triangular ox-pen as in the following diagram:
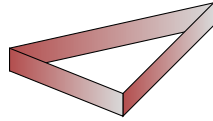
She has 2021 thin pieces of pre-cut timber, the smallest of size 200×1 (height times width, both measured in centimetres), the next smallest of size 200×2, the next smallest of size 200×3, and so on, with the largest of size 200×2021. She selects three pieces of timber to build the ox-pen. If the smallest piece has size 200×1001, in how many different ways can she select the remaining two pieces?
You may use that $1 + 2 + 3 + \cdots + k = k(k+1)/2$.

6. Escape room. You and three of your friends are locked in an escape room. As part of the game you have all been given a number from 1 to 4, such that no two of you have the same number. In an adjoining room are four boxes numbered 1 to 4. Hidden inside each box is a key in the shape of one of the numbers 1 to 4. The keys, no two of which are the same number, have been placed in the boxes at random. Without opening a box you cannot see which key it contains. One at the time you can enter the second room and open (and then close) three of the four boxes. If all four of you manage to open a box containing a key matching your own number (do not take the key out!) you can all escape. You are *only* allowed to talk before the first person enters the second room. Show that if the four of you are clever you have a better than 40% chance of escaping.