

**THE UNIVERSITY OF MELBOURNE - BHP BILLITON SCHOOL
MATHEMATICS COMPETITION 2006**

JUNIOR DIVISION

Time Allowed: TWO hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately.

The following suggestions are made for your guidance:

- 1. The examiners will attach great weight to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant solution than for a clumsy solution.*
- 2. The questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*
- 3. It may be necessary to spend considerable time on a problem before any real progress is made.*
- 4. You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.*
- 5. Credit will be given for partial solutions; however, a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks are **NOT** allowed. Electronic calculators, tables, etc., may be used. Computers may not be used. Calculators capable of storing text should have their memories erased before use. Otherwise, normal examination conditions apply.*

Warning: *Make sure you have the correct problems (Senior, Intermediate or Junior) in front of you.*

1. Sudoku. Place the numbers 1, 2, 3 and 4 in the Sudoku square

		1	
4		3	
2			

such that each number appears exactly once

- (1) in each row,
- (2) in each column,
- (3) in each of the four 2 by 2 blocks.

Explain your reasoning for each number entered.

2. Number crunch. Five different numbers are given. By computing all the different sums of 2 numbers we get the list

8, 11, 13, 14, 15, 16, 18, 19, 21,

where, possibly, some of the numbers in the list have occurred more than once. Find the 5 numbers.

3. Tennis woes. Bec and Leyton are building their dream home in the hills around Adelaide. Because they cannot remember the size of a regular tennis court they call their friend Mark Ph. in Melbourne. Mark has just broken up with his girlfriend and is really cranky. All he is willing to give away is that

- (a) tennis courts are rectangular,
- (b) if folded along the line of the net, the new court is similar* to the old court,
- (c) the area of the court is 200 square metres.

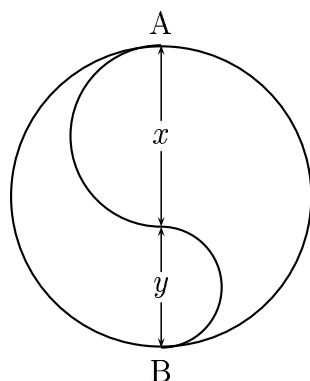
Bec and Leyton are very confused by all this. Can you help them out, and compute the dimensions of a tennis court?

* Similar means “of the same shape”, i.e., the ratio of the long and short sides of similar rectangles is the same.

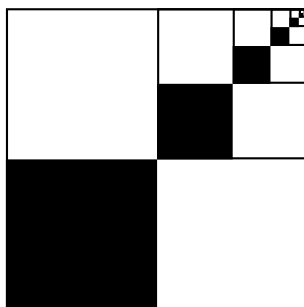
4. Squares and their divisors. Any positive integer d that divides a positive integer n is called a divisor (or factor) of n . For example, the numbers 1, 2, 4, 5, 10, 20 are divisors of 20, and there are no other divisors of 20.

- (a) Show that if a positive integer n is not a perfect square (that is, not one of 1, 4, 9, 16, 25, ...) it has an even number of divisors.
- (b) Show that if a positive integer n is a perfect square it has an odd number of divisors.

5. Yin and Yang. The area of a circle of diameter d is given by $\pi d^2/4$. Find the ratio of the areas of the left and right sides of the following Yin-Yang symbol. Note that the entire picture is made up of four semicircles (2 of which form the large full circle) whose centres are all on the same straight line from A to B.



6. Squares. Use the following figure



to show that

$$\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots = \frac{1}{3}.$$

7. Footy colours. The great wizard of Oz promises a group of 6 Carlton, 8 Collingwood and 10 Cats supporters that if they succeed in the following game, one of their teams will win the flag in 2006. (Which one is to be decided at a later stage and not by the outcome of the present game.)

The rules of the game are as follows. Each of the supporters starts out wearing their club jumper. When they meet a supporter from another club they both take off their jumper and put on a jumper of the third team. So, if someone wearing a Collingwood jumper meets someone wearing a Cats jumper, they both put on a Carlton jumper, if someone wearing a Collingwood jumper meets someone wearing a Carlton jumper, they both put on a Cats jumper, and if someone wearing a Cats jumper meets someone wearing a Carlton jumper, they both put on a Collingwood jumper. The aim of the game is to keep changing jumpers till all 24 footy fans wear the same jumper.

Assuming that the supporters are really clever (almost as clever as you perhaps) show either (i) why one of the above teams will win the 2006 grand final (by giving a winning strategy for the game), or (ii) why none of the above teams will win the flag in 2006.

Note: All supporters are desperate enough for their team to win the flag that they don't mind wearing each other's jumpers. If you have trouble imagining really clever C... supporters you may try the game with your own three favourite footy teams.