The University of Melbourne–Department of Mathematics and Statistics
School Mathematics Competition, 2019
JUNIOR DIVISION
Time allowed: Two hours

These questions are designed to test your ability to analyse a problem and to express yourself clearly and accurately. The following suggestions are made for your guidance:

(1) *Considerable weight will be attached by the examiners to the method of presentation of a solution. Candidates should state as clearly as they can the reasoning by which they arrived at their results. In addition, more credit will be given for an elegant than for a clumsy solution.*

(2) *The **six** questions are not of equal length or difficulty. Generally, the later questions are more difficult than the earlier questions.*

(3) *It may be necessary to spend considerable time on a problem before any real progress is made.*

(4) *You may need to do considerable rough work but you should then write out your final solution neatly, stating your arguments carefully.*

(5) *Credit will be given for partial solutions; however a good answer to one question will normally gain you more credit than sketchy attempts at several questions.*

*Textbooks, electronic calculators and computers are **NOT** allowed. Otherwise normal examination conditions apply.*

1. Year of the pig. Pigs are known to be highly intelligent animals. In the 1780s, Samuel Bisset's 'Learned Pig' could spell words, do basic arithmetic and tell the time. In 1968, the Yippies (Youth International Party) were so convinced of humanity's inferior intellect compared to that of pigs that they elected a pig named Pigasus as their party leader. As more recent proof of pigs' amazing abilities, on YouTube you can watch Moritz the Pig do jigsaw puzzles. Solve the following word puzzle by assigning to each of the eight letters P, I, G, S, M, A, R, T one of the numbers $1, 2, \ldots, 8$ such that different letters are assigned a different number, and such that $P = 5$:

$$\begin{array}{r} \text{P I G S} \\ \text{P I G S} \\ \underline{\text{P I G S}} \\ \text{S M A R T} \end{array} +$$

2. Penrose stairs. From 2 December 2018 till 7 April 2019 the National Gallery of Victoria featured the exhibition *Escher X Nendo: Between Two Worlds* showcasing works of Dutch graphical artist M. C. Escher (1898–1972). One of the key pieces on display was the lithograph *Ascending and Descending*, showing 26 monk-like men ascending or descending what are known as *Penrose stairs*.
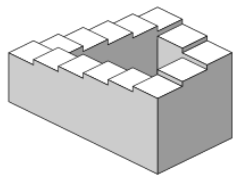
Starting on the same step of the 14-step Penrose stairs shown on the left, two monks start walking in opposite direction. The monk who is descending has a much easier time and walks twice as fast as the monk ascending. After how many combined steps do the monks meet for the 12th time (excluding their meeting at the start)?
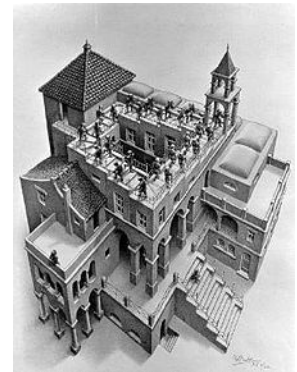
3. Super netball. The 2019 Australian Super Netball Competition kicked off four days ago, on the 27th of April. The top two teams of the regular season will play a best-of-five finals series, with the first team to win 3 games winning the flag. If the Melbourne Vixens play the NSW Swifts in the finals, and if both teams are equally matched (i.e., equally likely to win any particular game), what is the probability that the Vixens will win the flag *after* they have won the first game of the series? (Note: In Super Netball there are no draws.)

4. Notre-Dame. The Notre-Dame de Paris is a prime example of French Gothic architecture and one of the most iconic cathedrals in the world, attracting over 12 million visitors each year. Construction of the Notre-Dame started in 1160 and took more than 100 years to complete. On 15 April of this year the Notre-Dame caught fire, destroying a large part of the roof including the cathedral's main spire.

To prevent the collapse of one of the fire-damaged main towers, 100-metres-tall vertical beams have been erected adjacent to the tower. To keep these in place, each 100-metres-tall beam is supported by a 60-metres-tall vertical beam, connected to the taller beam by two diagonal beams. At what height do these diagonal beams cross?
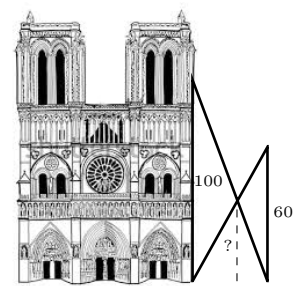
5. Going to the polls. Each candidate running for the seat of Parkville at the upcoming 18 May Federal election either likes to eat lamingtons or raw onions, but not both. The average age of the group of candidates who like lamingtons is 30 and the average age of the group of candidates who like raw onions is 50. If electoral hopeful Sir Les Patterson were to change his preference from raw onions to lamingtons, the average age of both groups would go up by 1 year. How many candidates are running for the seat of Parkville?

6. One could not make this up. On the topic of elections, on 21 April this year, Ukraine elected comedian Volodymyr Zelenskiy as its next President. Zelenskiy rose to fame in the Ukraine for playing the character of Vasyl Petrovych Holoborodko in the political satire *Servant of the people*, in which Holoborodko, a high-school history teacher, is unexpectedly elected as President-for-life of the Ukraine. As President, Holoborodko claims to be incorruptible, unlike past Ukrainian Presidents, while at the same time making many unrealistic promises. In his early-morning radio address to the nation, which he has been making daily since the 10th anniversary of his presidency, he each day promises "Today, I will solve more of Ukraine's problems than I solved two days ago but fewer than I solved 2019 days ago." For how many days can Holoborodko be true to his word?