1. Solutions

1. Year of the pig. Since P = 5 we must have S = 1 and hence also T = 3:

$$\begin{array}{rrrrrl} & 5 & \mathrm{I} & \mathrm{G} & 1 & \\ & 5 & \mathrm{I} & \mathrm{G} & 1 & \\ & 5 & \mathrm{I} & \mathrm{G} & 1 & \\ \hline 1 & \mathrm{M} & \mathrm{A} & \mathrm{R} & 3 & + \end{array}$$

The only odd number not yet assigned is 7. Since three times an odd number is an odd number, G and hence also R must be even. Moreover, since $3 \times 5 = 15$ it follows that M must be 6 or 7. Assume now that M = 6. Then I cannot be one of 7 or 8, so that A would have to be 7. This is only possible if I = 2. But

$$\begin{array}{rrrrrl} & 5 & 2 & \mathrm{G} & 1 & \\ & 5 & 2 & \mathrm{G} & 1 & \\ & 5 & 2 & \mathrm{G} & 1 & \\ \hline 1 & 6 & 7 & \mathrm{R} & 3 & + \end{array}$$

Since it is impossible to correctly complete the above with the remaining digits 4 and 8. Hence our assumption was wrong and M = 7:

$$\begin{array}{rrrrrl} & 5 & \mathrm{I} & \mathrm{G} & 1 & \\ & 5 & \mathrm{I} & \mathrm{G} & 1 & \\ & 5 & \mathrm{I} & \mathrm{G} & 1 & \\ \hline 1 & 7 & \mathrm{A} & \mathrm{R} & 3 & + \end{array}$$

It is now not hard to fill in the remaining even numbers, leading to

$$\begin{array}{rrrrrl} & 5 & 8 & 2 & 1 & \\ & 5 & 8 & 2 & 1 & \\ & 5 & 8 & 2 & 1 & \\ \hline 1 & 7 & 4 & 6 & 3 & + \end{array}$$

A harder problem is to show that if P $\neq$ 5 then there are no solutions.

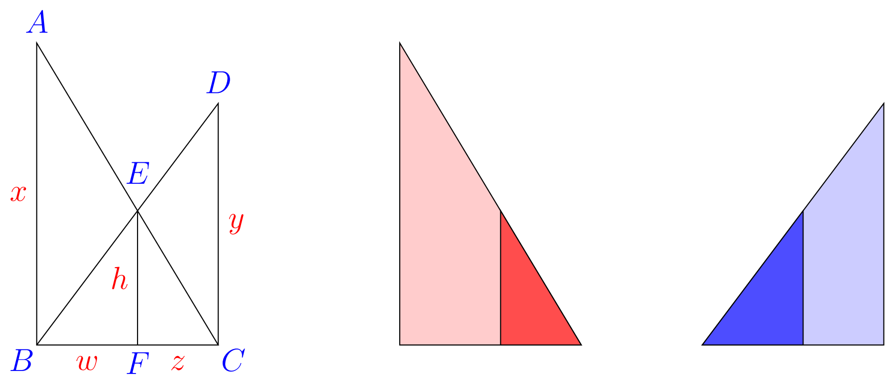
2. Penrose stairs. After 14 steps the ascending monk is back at the start. In this same time the descending monk has taken 28 steps and is back at the start for the second time. Hence, after a combined 42 steps, the monks meet up for the third time. (The first meeting is when the ascending monk has done a 1/3 of lap and the descending monk has done 2/3 of a lap, the second meeting is when the ascending monk has done 2/3 of a lap and the descending monk has done 4/3 laps.) Repeating this four times, it follows that after $4 \times 42 = 168$ combined steps they meet for the 12th time.
There is another way to solve the problem as follows. Say, for convenience, that monk A ascends at a speed of 1 step per second and monk D descends at a speed of 2 steps per second. We are interested in when they meet for the 12th time. But this is the same time they would have met for the 12th time if A were standing still and monk D were descending at 3 steps per second. Hence they meet for a 12th time after $(14 \times 12)/3 = 56$ seconds. The combined number of steps they have taken in this time is $56 \times 1 + 56 \times 2 = 56 \times 3 = 168$.

3. Super netball. After the first game won by the Vixens there are 4 more games to be played. Each game has two possible outcomes giving a sequence of $2 \times 2 \times 2 \times 2 = 16$ possible outcomes, each being equally likely. There are only 5 outcomes that will lead to the Swifts winning the flag:

$$ssss, sssv, ssvs, svss, vsss.$$

Hence the Swifts have a $5/16 \times 100\% = 31.25\%$ chance of winning, so that the Vixens have a $11/16 \times 100\% = 68.75\%$ chance of winning the flag. (The final game of *ssss* and *sssv* will not actually be played, as the flag goes to the first team to win three games, but this does not affect the counting argument; the probability of games 2, 3 and 4 all being won by the swifts is 1/8 which is the same as 1/16 for *ssss* plus 1/16 for *sssv*.)

4. Notre-Dame. We want to find h (in metres) given that $x = 100$ metres and $y = 60$ metres:



Since the triangles ABC and EFC are *similar*,

(1) $$\frac{x}{w+z} = \frac{h}{z}.$$

Likewise, since the triangles BCD and BFE are similar triangles,

$$\frac{w+z}{y} = \frac{w}{h}.$$

Multiplying these two equations yields

$$\frac{x}{y} = \frac{w}{z}.$$

But (1) may be rewritten as

$$h = \frac{xz}{w+z} = \frac{x}{w/z+1} = \frac{x}{1+x/y} = \frac{xy}{x+y}.$$

Interestingly, this does not depend on $w+z$ If $x = 100$ and $y = 60$ then $h = 60 \times 100/(100+60) = 75/2 = 37.5$ metres. Interestingly, this does not depend on the distance between the two vertical beams.

5. Going to the polls. Let there be ℓ lamington lovers and u onion lovers among the candidates. The problem is to find $\ell + u$, which gives the total number of candidates. The combined age of lamington lovers is 30ℓ and the combined age of onion lovers is $50u$. If Sir Les were to change his preference this would change to $31(\ell + 1)$ and $51(u - 1)$, respectively. Since the combined age of all candidates is fixed, we must have $30\ell + 50u = 31(\ell + 1) + 51(u - 1)$. This implies that $\ell + u = 20$. Hence there are 20 candidates running for the seat of Parkville.

6. One could not make this up. Temporarily drop the "but fewer than I solved 2019 days ago" part of Holoborodko's promise, and assume that two days before he started broadcasting he solved x problems and one day before his first broadcast he solved y problems. The minimum required effort to then be true to his word (for ever!) would lead to the following sequence:

$$\underbrace{x, y}_{\text{before broadcasting}} \quad | \underbrace{x+1, y+1, x+2, y+2, \ldots}_{\text{after broadcasting}},$$

where Holoborodko solves $x+1$ problems on day one of broadcasting, $y+1$ problems on day two and so on (and y problems on day "0" and x problems on day "-1"). Now also include "but fewer than I solved $2n+1$ days ago" (so that in the actual problem $n = 1009$). This means that on day $-2n$ he should have solved at least $x+2$ problems, on day $-(2n-1)$ at least $y+2$ problems, on day $-(2n-2)$ at least $x+3$ problems, on day $-(2n-3)$ at least $y+3$ problems, and so on. In other words, on day $-(2n-2i+2)$ he needs to have solved at least $x+i+1$ problems and on day $-(2n-2i+1)$ at least $y+j+1$ problems. To do this for as long as possible we would like i to get as large as possible. If we try $i = n$ then it says that on day -2 at least $x+n+1$ and on day -1 at least $y+n+1$ problems need to be solved. But we already know that on day -1 a total of x problems were solved. Hence we require $y = x-n-1$, leading to

$$\underbrace{x+2, x-n+1, x+3, x-n+2, \ldots, x+n+1, x, x-n-1}_{2n+1 \text{ days}}$$

$$| \underbrace{x+1, x-n, x+2, x-n+1, \ldots, x+n, x-1}_{2n \text{ days}}.$$

Trying $i = n+1$ does, however, not work. It would imply that on day 0 Holoborodko solves $y = x-n-1$ but also $x+n+2$ problems. This clearly is impossible. In the above sequence we may without loss of generality choose $x = n+1$, which avoids Holoborodko solving a negative number of problems (i.e., creating problems instead) on any given day. An optimal solution is thus

$$\underbrace{n+3, 2, n+4, 3, \ldots, 2n+2, n+1, 0}_{2n+1 \text{ days}} | \underbrace{n+2, 1, n+3, 2, \ldots, 2n+1, n}_{2n \text{ days}}$$

so that the answer is $2n$ days, in our case 2018 days.